Estimation of option's parameters based on particle swarm optimization algorithm

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Abstract: As a landmark in contingent claim theory, Black-Scholes model has been widely used in financial markets. However, these exists a difficulty that some important variables should be estimated when applying the model to price options. The more accurately these parameters are estimated, the more accurate approximate values of option prices will be. In order to seek estimates of these parameters, an improved particle swarm optimization algorithm is considered, in which the mutation operation is made on particles past optimal and global optimal positions. Then we apply the algorithm to obtain the approximations of volatility and risk-free rate in European option model. Compared with the binary particle swarm optimization algorithm with bit change mutation, our algorithm is better in stability and convergence speed.

1. Introduction

Option markets are the most popular shares of financial institution, and option pricing is considered the most complex mathematically in these applied areas of finance. In 1973, analytical expressions for European options were given by Black, Scholes and Merton. After Black-Scholes model was proposed, plenty of research results about applications and developments of BS model have emerged. Minster and Koehler [1] applied the Newton-Raphson method to analyze the volatility of option. Avellaneda et al. [2] studied how to price different derivative securities when market volatilities were uncertain. Dindar and Marwala [3] proposed an optimized networks to deal with option pricing better. In fact, it's impossible to gain the analytical expressions for some kinds of options in real world. And numerical algorithms have been developed to deal with the computational intractability gradually. In the past few decades, some heuristic algorithms (such as, genetic algorithm (GA) [4], ant colony optimization (ACO) [5], normalized particle swarm optimization [6], and artificial glowworm swarm optimization [7]) have been applied to price options, and these methods displayed more flexibility and capacity compared with Black-Scholes model.

It's worth noting that these parameters in BS model were set to be constants in most research on option pricing. Actually, option price is very sensitive to the parameter's fluctuation. Meanwhile, these parameters such as volatility and riskless interest rate can't be observed in advance, and the settings of option need to be estimated based on historical data. Thus it's critical to price options that how to choose proper parameters by a class of numerical algorithms. Since there exist complicated nonlinear relations among these parameters, we consider particle swarm optimization algorithm (PSO) to overcome the difficulty.

As a class of evolutionary computation methods, particle swarm optimization algorithm (PSO) was developed by Kennedy and Eberhart [8]. The procedure of original PSO algorithm is prone to code and implement, meanwhile the algorithm tends to premature. And enormous research on modification and application about PSO algorithm have been undertaken. For example, in order to enhance particle's exploit ability, He and Huang [12] used the mean value of past best positions to renew particle's position in PSO algorithm, Liu et al. [13] considered particle swarm optimizer with constraint. Garg [14] applied a hybrid PSO-GA algorithm to solve a class of constrained optimization problems. Meanwhile, research subjects about applying PSO to parameter estimation of option have been carried out. Based on the idea of the discrete binary version of particle swarm algorithm [9], Lee et al. [10] made bit change mutation on the algorithm and applied it to estimate the volatility of European option. Then Zhao et al. [11] considered quantum-behaved PSO algorithm, and used it to approximate the volatility of option.

This paper is organized as follows. Firstly, European option pricing model is introduced and the optimization problem about these parameters is proposed. Next, we design an improved particle swarm optimization algorithm, in which the mutation operation is made on particles past optimal and global optimal positions. Then, we use the improved algorithm to estimate these parameters of call option. Furthermore, we compare the improved algorithm with existed binary particle swarm optimization algorithms under different selections about the number of particle swarm and the maximal iteration. Finally, some important conclusions are given.

2. Problem Description

By solving their differential equation, Black and Scholes have obtained exact formulas for prices of European call and put options under necessary hypotheses.

- The stock price follows geometric Brownian motion;
- Options have to be exercised at the expiration date;
- The interest rate is constant;
- The stocks are traded continuously without dividends and tax;
- The market is considered to be frictionless.

Next we consider the European call option only. A stock is given whose price today is S_0. The value of a call option on this stock is denoted by V, and strike price, time to expiration, volatility of stock and riskless interest rate are denoted by and r respectively. And Black-Scholes model is given by X, τ, σ the following formula:

$$V = S_0 N(d_1) - Xe^{-r\tau} N(d_2),$$

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, d_2 = d_1 - \sigma\sqrt{\tau}.$$

In this formula, denotes the standard normal distribution function.

In Black-Scholes model, the volatility and risk-free rate are usually set to be constants during the period of analysis. Actually, different results of these parameters are obtained when we consider non-overlapping data and various time series. Therefore, it's necessary to look behind the estimates, then concern whether these values are higher or lower than we would expect from past and current values, and whether options more expensive or cheaper than they could be. In order to meet the feature of real financial market better, these parameters are set to be not constants, but variables which could be estimated by experiment analysis.

For finding better estimation of these parameters in BS model, we expect that the approximate price of the call option is the same as the actual option price. Thus, to measure the difference between estimated and actual call option value, we choose the sum of absolute deviation as the fitness function. Our objective is to minimize the difference. The related optimization problem is as follows.

$$\begin{aligned} \min \quad & \left| f(x_1, x_2) - f_0 \right| \\ s.t. \quad & 0 < x_i < 1, i = 1, 2 \end{aligned} \qquad (Q)$$

Where $f(x_1, x_2)$ denotes European option pricing formula, and represent the volatility x_1 and x_2 risk-free rate respectively, and f_0 is considered to be the real option price.

3. Particle Swarm Optimization Algorithm with some improvements

In PSO algorithm, set the dimension of search space to M dimensional, and number of particle swarm ton. And let the position and velocity of ith particle be

$X_i = (X_{i1}, X_{i2}, \cdots, X_{iM}), V_i = (V_{i1}, V_{i2}, \cdots, V_{iM})$.

The past optimal position and global optimal position of ith particle are set to be

$P_i = (P_{i1}, P_{i2}, \cdots, P_{iM})$, $P_g = (P_{g1}, P_{g2}, \cdots, P_{gM})$.

In the process of iteration, the velocity of particle and its position will be renewed by the following equations:

$$V_{ik}(t+1) = wV_{ik}(t) + c_1 r_{1i,k}(t)(P_{ik}(t) - X_{ik}(t)) + c_2 r_{2i,k}(t)(P_{gk}(t) - X_{ik}(t)) \quad (1)$$

$$X_{ik}(t+1) = X_{ik}(t) + V_{ik}(t+1) \quad (2)$$

Where $i = 1, 2, \cdots, n, k = 1, 2, \cdots, M$, w is the inertia weight, c_1 and c_2 are cognitive and social scaling parameters respectively, $r_{1i,k}$ and $r_{2i,k}$ are uniformly distributed in the interval (0,1).

Some improvements on PSO algorithm are considered, and the improved algorithm is denoted by IPSO. In the algorithm, assume $c_1 = c_2 = 2$ and set was follows:

$$w(t) = w_{\min} + \frac{T - t}{T - 1}(w_{\max} - w_{\min})$$

Where T and t are maximal and present iteration counts respectively, and $w_{\max}$ and $w_{\min}$ are maximal and minimal values of inertia weight respectively. In order to exploit particle's search ability, replace the past best position by the mean of the past best positions. For increasing particle's diversity, mutation operations are made on the global and past best positions.

The procedure flow of the improved algorithm is as follows:

- Initialize all particles, and set iteration count to 0; Calculate the fitness value of every particle.
- Replace the current particle's position while its fitness value is worse than the values of past optimal position; Meanwhile, replace particle's position with global best position.
- Renew global optimal position and past optimal position by mutation operations
- Renew iteration velocity and position by (1) and (2), and $t = t + 1$.
- Turn to Step 2 while t is less than the maximal iteration count, unless end the process.

4. Results and discussion

In the experiment, we choose the binary particle swarm optimization algorithm (BPSO1), the binary particle swarm optimization algorithm with bit change mutation (BPSO2) and IPSO to solve problem (Q) respectively. These data in the experiment come from [15]. In numerical experiments, a 30 dimensional binary code is chosen in the binary swarm optimization algorithm, and a 1 dimensional real value code is used in our improved algorithm. Then we consider three different selections about the number of particle swarm and the maximal iteration count. After repeat the experiment thirty times, record the average of results.

Table 1. Comparison of the Experimental Results of the Three Algorithms

Selection	n	T	BPSO1	BPSO2	IPSO
S1	10	50	0.1709	0.1335	0.0040
S2	20	50	0.0738	0.0819	0.0012
S3	20	100	0.0392	0.0297	5.9138E-004

These numerical results of different algorithms are showed in Table 1. Compared with BPSO1 and BPSO2, the improved algorithm IPSO achieves better results and performs stronger searching ability in three selections. In the system of the improved particle swarm algorithm, we take past optimal position and global optimal position of every particle into account, meanwhile apply mutation operation to increase particle's diversity. It's because of this, particle's exploit ability is enhanced.

Table 2. Fitness comparison between BPSO2 and IPSO

Selection	Algorithm	Min	Max	Mean	Std. Dev.
S1	BPSO2	0.0033	0.3336	0.1335	0.0091
	IPSO	1.7617E-007	0.0212	0.0040	2.1549E-005
S2	BPSO2	0.0024	0.2863	0.0819	0.0058
	IPSO	1.9033E-005	0.0040	0.0012	1.3801E-006
S3	BPSO2	0.0028	0.1103	0.0297	7.0510E-004
	IPSO	7.4317E-006	0.0045	5.9138E-004	9.6660E-007

Table 3. Volatility estimate values of different algorithms

Selection	BPSO1	BPSO2	IPSO
S1	0.2036	0.2570	0.2964
S2	0.2345	0.2503	0.3189
S3	0.3216	0.3282	0.3567

Table 4. Risk-free rate estimate values of different algorithms

Selection	BPSO1	BPSO2	IPSO
S1	0.1505	0.1167	0.1103
S2	0.1387	0.1359	0.0906
S3	0.0714	0.0806	0.0597

Furthermore, estimation results of volatility and risk-free rate by using different algorithms are demonstrated in Table III and Table IV respectively. While minimizing the difference between estimated and actual call option value, the estimation values of parameters by IPSO are closer to the real ones. Among three selections, when the number of particle is 20 and iteration count is 100, IPSO display stronger computing efficiency.

5. Conclusion

In order to obtain the numerical solutions for parameters of call option, an improved particle swarm optimization algorithm (IPSO) is proposed. For avoid premature, we optimize past optimal position and global optimal position in particle swarm system. In numerical experiments, it's easier for IPSO to escape local optimum and approach global optimum, and IPSO is better in stability and convergence speed. Compared with binary particle swarm optimization algorithms, the estimation values of parameters by IPSO are closer to the real ones.

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